

Kaonic hydrogen atom and K^-p scattering length

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Kaonic hydrogen is studied with various realistic potentials in an accurate numerical approach based on Sturmian functions. The K^-p scattering length extracted from the $1s$ energy shift of the kaonic hydrogen by applying the Deser-Trueman formula is severely inconsistent with the one derived by directly solving the scattering Schrödinger equation. We pay special attention to the recent measurement of the energy shift and decay width of the $1s$ kaonic hydrogen state by the DEAR Collaboration. After taking into account the large discrepancy between the extracted and directly-evaluated scattering lengths, we found theoretical predictions of most chiral SU(3) based models for the kaonic hydrogen decay width are consistent with the DEAR data. We warn the SIDDHARTA collaboration that it may not be reasonable to extract kaon-nucleon scattering lengths, by using the Coulomb-interaction corrected Deser-Trueman formula, from the planned measurement of kaonic hydrogen.

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Kaonic hydrogen is mainly the Coulomb bound state of a K^- and a proton but is affected by the strong interaction at small distances. Furthermore, the strong interaction couples the K^-p state to the \bar{K}^0n , $\pi\Sigma$, $\pi\Lambda$, $\eta\Sigma$ and $\eta\Lambda$ channels and results in the $\pi\Sigma$ and $\pi\Lambda$ decaying modes. It is believed that the study of kaonic hydrogen effectively probes the low-energy, especially zero energy strong kaon-nucleon interaction. Inspired by the recent precise determination of the energy and decay width by the DEAR Collaboration [1], kaonic hydrogen has been extensively studied in the theoretical sector, mainly in effective field theory [2, 3, 4, 5, 6, 7, 8, 9, 10]. Theoretical predictions for the K^-p scattering length have been compared with the DEAR data of the energy and decay width of kaonic hydrogen, by using the Deser-Trueman formula [11, 12]

$$\Delta E_{1s} + i \frac{\Gamma_{1s}}{2} = 2\alpha^3 \mu^2 a_p \quad (1)$$

or the Coulomb-interaction corrected Deser-Trueman formula [5]

$$\Delta E_{1s} + i \frac{\Gamma_{1s}}{2} = 2\alpha^3 \mu^2 a_p [1 - 2\alpha \mu a_p (\ln \alpha - 1)] \quad (2)$$

where μ is the reduced mass of the K^-p system, ΔE_{1s}

and Γ_{1s} are the energy shift and decay width of the $1s$ kaonic hydrogen due to the strong interaction, and a_p stands for the S-wave K^-p scattering length. A general result is that the energy shifts and decay widths extracted, by using eq. (1) or (2), from theoretical K^-p scattering lengths are much larger than the DEAR data.

In this work we show that eqs. (1) and (2) may not well hold for the $\bar{K}N$ system and the discrepancy between the theoretical results and the DEAR data is not that large and particularly the theoretical decay widths are consistent with the DEAR data.

We derive the K^-p scattering length and kaonic hydrogen energy shift and decay width by solving the same dynamical equation,

$$\left[-\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d}{dr} \right) + \frac{l(l+1)}{r^2} - \mathbf{Q}^2 + \mathbf{f} \cdot \mathbf{V} \right] \mathbf{R}(r) \quad (3)$$

with

$$\mathbf{Q}^2 = \begin{pmatrix} q_c^2 & 0 \\ 0 & q_0^2 \end{pmatrix}, \quad \mathbf{f} = \begin{pmatrix} f_c & 0 \\ 0 & f_0 \end{pmatrix}, \quad (4)$$

$$\mathbf{V} = \mathbf{V}^{em} + \mathbf{V}^h, \quad (5)$$

$$\mathbf{V}^{em} = \begin{pmatrix} V^{em} & 0 \\ 0 & 0 \end{pmatrix}, \quad (6)$$

$$\mathbf{V}^h = \begin{pmatrix} \frac{1}{2}(V_1^h + V_0^h) & \frac{1}{2}(V_1^h - V_0^h) \\ \frac{1}{2}(V_1^h - V_0^h) & \frac{1}{2}(V_1^h + V_0^h) \end{pmatrix}, \quad (7)$$

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$$\mathbf{R}(r) = \begin{pmatrix} R_{K^-p}(r) \\ R_{\bar{K}^0n}(r) \end{pmatrix}, \quad (8)$$

$$q_c^2 = \frac{[E^2 - (M_p - M_{K^-})^2][E^2 - (M_p + M_{K^-})^2]}{4E^2}, \quad (9)$$

$$q_0^2 = \frac{[E^2 - (M_n - M_{\bar{K}^0})^2][E^2 - (M_n + M_{\bar{K}^0})^2]}{4E^2}, \quad (10)$$

$$f_c = \frac{E^2 - M_p^2 - M_{K^-}^2}{E}, \quad (11)$$

$$f_0 = \frac{E^2 - M_n^2 - M_{\bar{K}^0}^2}{E} \quad (12)$$

where V^{em} is the electromagnetic potential, V_0^h and V_1^h are respectively the isospin $I=0$ and 1 strong interactions of the $\bar{K}N$ system, $R_{K^-p}(r)$ and $R_{\bar{K}^0n}(r)$ are respectively the K^-p and \bar{K}^0n parts of the radial wave function of the $\bar{K}N$ system. Eq. (3) embeds into the Schrödinger equation the relativistic effect and the mass difference between the K^-p and \bar{K}^0n components. The relativistic modification of the Schrödinger equation to eq.(3) has been discussed in the works [13, 14, 15, 16, 17, 18].

We study kaonic hydrogen and kaon-nucleon scattering with the phenomenological $\bar{K}N$ potential taken from the work [19, 20] and the various effective potentials which are worked out in the work [21]. The interaction [19, 20] is constructed by fitting the free $\bar{K}N$ scattering data [22], the KpX data of kaonic hydrogen by the KEK Collaboration [23] and the binding energy and decay width of $\Lambda(1405)$, which is regarded as an isospin $I = 0$ bound state of $\bar{K}N$. Since the interaction gives one molecular state $\Lambda(1405)$, it must be much stronger than the strong pion-pion interaction.

In the work [21] an effective local potential in coordinate space is constructed such as the solution of the Schrödinger or Lippmann-Schwinger equation with such a potential approximates as closely as possible the scattering amplitude derived from the full chiral coupled-channel calculation. Several realistic chiral SU(3) based models [3, 4, 8, 10] have been studied.

The accurate evaluation of energy shifts, decay widths and especially wave functions of exotic atoms has been a challenge to numerical methods [17, 24]. An approach is required, which is able to account accurately for both

TABLE I: Energy shift ΔE_{1s} , decay width Γ_{1s} of kaonic hydrogen and K^-p scattering length a_{K^-p} are derived by directly solving the dynamical equation eq. (3). \tilde{a}_{K^-p} are extracted from the energy shifts and decay widths in Column 1 and 2 by applying the Deser-Trueman formula of eq. (1). Energy shifts and decay widths are given in eV.

$V_{\bar{K}N}(r)$	ΔE_{1s}	$\Gamma_{1s}/2$	a_{K^-p} [fm]	\tilde{a}_{K^-p} [fm]
AY [20]	-384	144	-1.012 + 0.499 <i>i</i>	-0.934 + 0.348 <i>i</i>
HNJH [4]	-336	324	-0.778 + 1.084 <i>i</i>	-0.815 + 0.785 <i>i</i>
ORB [3]	-348	323	-0.804 + 1.067 <i>i</i>	-0.845 + 0.784 <i>i</i>
BNW [8]	-288	337	-0.625 + 1.068 <i>i</i>	-0.700 + 0.818 <i>i</i>
BMN [10]	-297	311	-0.655 + 0.992 <i>i</i>	-0.721 + 0.755 <i>i</i>

the strong short-range interaction and the long-range Coulomb force. The numerical approach based on Sturmian functions [25] has been found effective and accurate. In this work we use the numerical method which has been carefully studied and discussed in [25, 26, 27] to study kaonic hydrogen.

Shown in Table I are our theoretical results with various realistic $\bar{K}N$ potentials. The energy shift ΔE_{1s} and decay width Γ_{1s} of the $1s$ kaonic hydrogen state are derived by solving eq. (3) in the above mentioned Sturmian function approach [25, 26, 27]. The negative energy shifts in Table I mean that the $1s$ energy level is effectively pushed up by the strong interaction since there exists one deep bound state, the $\Lambda(1405)$. The K^-p scattering lengths a_{K^-p} in Table I are directly evaluated from eq. (3). Listed in the last column of Table I are the extracted K^-p scattering length \tilde{a}_{K^-p} from the energy shifts ΔE_{1s} and decay width Γ_{1s} in Column 1 and 2 by applying the Deser-Trueman formula of eq. (1).

It is clear that with the same interaction [20, 21] the scattering length derived by directly solving the Schrödinger equation in eq. (3) is rather different from the one extracted from the energy shift and decay width of the $1s$ kaonic hydrogen by applying the Deser-Trueman formula of eq. (1). One finds from Table I that for the imaginary part the extracted scattering length is smaller by a factor of 20% to 30% than the directly-derived scattering length with the same interaction. Averaging over the results for all the five potentials we get the averaged factor to be 0.26. The result implies that both the lowest Deser-Trueman formula in eq. (1) and the Coulomb interaction corrected one in eq. (2) may not well apply

to the $\bar{K}N$ system. The Coulomb interaction leads to a correction, less than 10%, to the lowest Deser-Trueman formula in eq. (1). Hence one may argue that for the $\bar{K}N$ system an extraction, by using the formulas in eqs. (1) or (2), may not be accurate, if not say, unreliable.

It has been puzzling that the energy shifts and decay widths, extracted from the scattering lengths derived in a number of chiral SU(3) based models [2, 8, 9, 10], are inconsistent with the DEAR data of kaonic hydrogen. However, we find, after considering the large discrepancy between the directly evaluated and the extracted scattering lengths, that the inconsistency between the theoretical decay width and the DEAR data is not that obvious.

Listed in the second and third columns of Table II are respectively the isospin 0 and 1 $\bar{K}N$ scattering lengths predicted by various chiral SU(3) based models [2, 6, 7, 8, 9, 10] where the isospin symmetry limit is applied. In the fourth column we give the corresponding K^-p scattering lengths evaluated from the isospin based scattering lengths a_0 and a_1 , by using the formula

$$a_{K^-p} = \frac{(a_0 + a_1)/2 + a_0 a_1 q_0}{1 + (a_0 + a_1) q_0/2} \quad (13)$$

with

$$q_0 = \sqrt{2\mu_0\Delta} \quad (14)$$

where μ_0 is the reduced mass of the \bar{K}^0n system and Δ the mass difference between the \bar{K}^0n and K^-p systems. The formula in eq. (13) holds with a high accuracy. We found a K^-p scattering length evaluated with the formula in eq. (13) has a difference about 1% from the one derived by directly solving the particle based dynamical equation in eq. (3) for the potentials listed in Table I. The decay widths Γ_{1s} in the last column of Table II are extracted from the scattering lengths listed in the fourth column with the formula

$$\frac{\Gamma_{1s}}{2} = 2\alpha^3 \mu^2 \text{Im} a_{K^-p} (1 - R) \quad (15)$$

with $R = 0.35$. We choose $R = 0.35$ based on the facts that the Coulomb interaction correction to the lowest Deser-Trueman formula is up to 10% and on average over the five realistic $\bar{K}N$ potentials the extracted K^-p scattering length with the lowest Deser-Trueman formula is smaller by a factor of 0.26 than the directly evaluated one.

TABLE II: Isospin based scattering lengths a_0 and a_1 are taken from the works listed in the table. a_{K^-p} are evaluated with the formula in eq. (13) and decay widths Γ_{1s} are extracted from the scattering lengths a_{K^-p} in Column 4 by using the modified Deser-Trueman formula in eq. (15). Experimental uncertainties of Martin's scattering data are not shown in the table.

Ref.	a_0 [fm]	a_1 [fm]	a_{K^-p} [fm]	$\Gamma_{1s}/2$ [eV]
[2]	$-1.31 + 1.24i$	$0.26 + 0.66i$	$-0.64 + 1.15i$	307
[8]	$-1.48 + 0.86i$	$0.57 + 0.83i$	$-0.78 + 0.95i$	254
[9]	$-1.23 + 0.45i$	$0.98 + 0.35i$	$-0.49 + 0.48i$	128
[10]	$-1.45 + 0.85i$	$0.65 + 0.76i$	$-0.74 + 0.93i$	248
[10]	$-1.72 + 0.77i$	$0.09 + 0.76i$	$-1.11 + 0.86i$	229
[10]	$-1.64 + 0.75i$	$-0.06 + 0.57i$	$-0.63 + 0.42i$	206
[6]	$-1.22 + 0.54i$	$0.26 + 0.00i$	$-0.78 + 0.60i$	112
[7]	$-1.50 + 0.66i$	$0.50 + 0.04i$	$-1.06 + 0.77i$	160
[22]	$-1.70 + 0.68i$	$0.37 + 0.60i$	$-1.03 + 0.76i$	202

Comparing to the DEAR data that

$$\frac{\Gamma_{1s}}{2} = 125 \pm 56 \text{ (stat)} \pm 15 \text{ (syst)} \text{ eV} \quad (16)$$

one finds from Table II that most decay widths extracted from the scattering lengths predicted by the chiral SU(3) based models are in line with the DEAR data of the 1s kaonic hydrogen decay width. We emphasize that the DEAR result of the 1s kaonic hydrogen decay width is well consistent with Martin's scattering data which leads, by applying the modified Deser-Trueman formula in eq. (15), to a decay width $\Gamma_{1s} = 202 \pm 20$ eV.

We conclude that for all the realistic local potentials employed in the work, the K^-p scattering length extracted from the energy shift and decay width of the 1s kaonic hydrogen by applying the lowest and Coulomb interaction corrected Deser-Trueman formulas is severely inconsistent with the one derived by directly solving the scattering Schrödinger equation. After taking into account the large discrepancy between the extracted and directly-evaluated scattering lengths, we found theoretical predictions for the kaonic hydrogen decay width by most chiral SU(3) based models are consistent with the DEAR data. We warn the SIDDHARTA collaboration that it may not be reasonable to extract kaon-nucleon scattering lengths, by using the Coulomb-interaction corrected Deser-Trueman formula, from the planned measurement of kaonic hydrogen.

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